Superconnexivity Reconsidered

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In my paper "Strong Connexivity" (cf. [1]), I claimed that the connexive principles were based on intuitions that made an amendment necessary. Connexive logic, up to that point, had largely been considered to be exhaustively characterized by the theses of Aristotle and Boethius[1]

ARISTOTLE: $\neg(A\to\neg A)$ and $\neg(\neg A\to A)$ are valid.

BOETHIUS: $(A\to B)\to\neg(A\to\neg B)$ and $(A\to\neg B)\to\neg(A\to B)$ are valid.

However, some connexive logics allowed for satisfiable instances of $(A\to\neg A)$, as well as simultaneously satisfiable instances of $(A\to B)$ and $(A\to\neg B)$. I took that to go against the spirit of the connexive enterprise. To be able to judge those cases out of bounds, I suggested to add two unsatisfiablity clauses:

UNSAT1: In no model, $(A\to\neg A)$ is satisfiable, and neither is $(\neg A\to A)$.

UNSAT2: In no model $(A\to B)$ and $(A\to\neg B)$ are satisfiable simultaneously (for any A and B).

To call for such clauses that prescribe unsatisfiabilities, of course, is a rather uncommon move, and I suggested, in a very tentative way, that there might be an idea worth exploring that tries to push the requirement into the object language. He wrote:

> In analogy to [the] use of explosion to express the unsatisfiability of any contradiction, we might try to ask that $(A\to\neg A)\to B$ should be valid, in order to express in the object language that $A\to\neg A$ is unsatisfiable (and similarly for the rest of the connexive theses). ([1, p.143])

I called a logic that satisfies these requirements *superconnexive*. However, adding this to a system with substitutivity of logical equivalents quickly leads to triviality.

[1] For an overview of connexive logic, see [2, 3].

In the paper I present, Hitoshi Omori and I explore whether something of the idea can be salvaged, even if we insist on keeping substitutivity. We argue that this is possible, even though we have to modify the idea a little bit.

Our idea is to replace the arbitrary B with a bottom, as follows:

SUPER-BOT-ARISTOTLE: $(A\rightarrow\neg A)\rightarrow\perp$ and $(\neg A\rightarrow A)\rightarrow\perp$ are valid.

SUPER-BOT-BOETHIUS: $(A\rightarrow B)\rightarrow((A\rightarrow\neg B)\rightarrow\perp)$ and $(A\rightarrow\neg B)\rightarrow((A\rightarrow B)\rightarrow\perp)$ are valid.

We will argue that by taming the behavior of $\perp$ just enough to retain a sense of absurdity but avoid triviality of the logical systems these principles are added to, we can make sense of the original super-connexive idea.

References

[1] Andreas Kapsner. Strong connexivity. *Thought*, 1(2):141–145, 2012.

[2] Storrs McCall. A History of Connexivity. In Dov Gabbay, editor, *Handbook of the History of Logic*, volume 11, pages 415–449. Elsevier, 2012.

[3] Heinrich Wansing. Connexive logic. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Spring 2020 edition, 2020.