# *Social Spheres: logic, ranking, and subordination*

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## *Subordination:*

1. ranking one person or a group of people below others,
2. depriving the lower-ranked of rights, and
3. legitimating discrimination against them.

Langton, R. (1993). Speech acts and unspeakable acts. *Philosophy and Public Affairs*, 22(4):293–330

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## *Spheres of Possible Worlds*

In Lewis' *Counterfactuals*, the actual world is just one among many worlds, $w_0, w_1, w_2$... etc., getting along in a modal space, $W$.

Lewis, D. (1973). *Counterfactuals*. Blackwell, Oxford

Each $w$ is associated with a *set of spheres* $\$_w$, which must meet the following four constraints:

C) Centering: $\{w\} \in \$_w$
1) Nesting: for $T, S \in S_w$, either $S \subseteq T$ or $T \subseteq S$
2) Closure under unions: if $\bigcup S$ is the union of a set of spheres in the system $\$_w$, then $\bigcup S \in \$_w$.
3) Closure under (non-empty) intersections: if $\bigcap S$ is the intersection of a set of spheres in the system $\$_w$, $\bigcap S \in \$_w$.

What justifies these constraints in *Counterfactuals* is the desire to use the $\$_w$ to represent overall comparative similarity to $w$.

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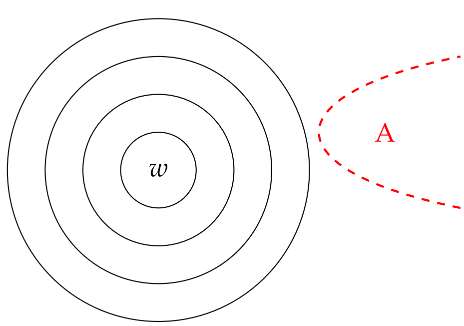
$A \;\Box\!\!\rightarrow B$ is true at a world $w$ (given $\$_w$, on $W$) if and only if either

1. no $A$-world belongs to any sphere $S$ in $\$_w$, or
2. some sphere $S$ in $\$_w$ contains at least one $A$-world, and $A \rightarrow B$ holds at every world in $S$.

← Here are the truth-conditions for counterfactuals

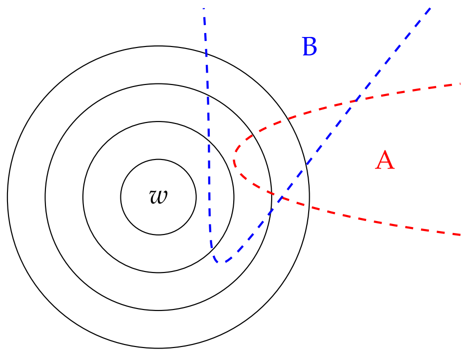
*Vacuous Truth: "If kangaroos squared the circle, they would fall over."*

$A \;\Box\!\!\rightarrow B$—true
$A \;\Box\!\!\rightarrow \neg B$—also true



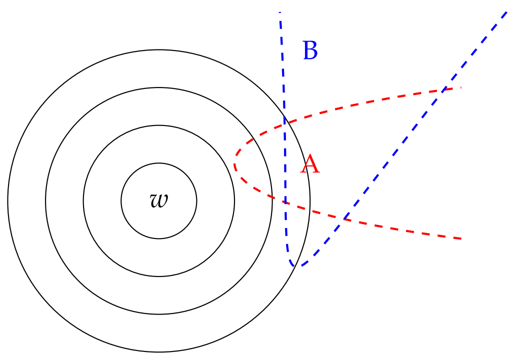
*Non-vacuous Truth: "If kangaroos had no tails, they would fall over."*

$A \mathrel{\Box\!\!\rightarrow} B$—true
$A \mathrel{\Box\!\!\rightarrow} \neg B$—false



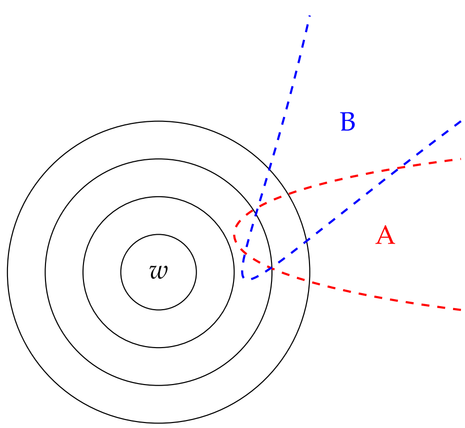

*Falsity; Opposite True: "If kangaroos had no tails, they would weigh more."*

$A \mathrel{\Box\!\!\rightarrow} B$—false
$A \mathrel{\Box\!\!\rightarrow} \neg B$—true




*Falsity; Opposite False: "If kangaroos had no tails, they would migrate eastwards.*

$A \mathrel{\Box\!\!\rightarrow} B$—false
$A \mathrel{\Box\!\!\rightarrow} \neg B$—false



## *Social Spheres*

1. Each of us is just one among many persons, $p_0, p_1, p_2, ...$ etc., getting along in a social space, $U$.
2. Suppose people are organised into a set of social spheres, $R$, where these spheres are intended to convey information about *social rank*.
3. The smaller the spheres of which an individual, $p$, is a member, the higher $p$'s rank.
4. Social rank (unlike comparative similarity) need not be relative to a third person; it's a binary rather than a ternary relation
5. So we need not think of our system of spheres as a function $\$_p$ from a person $p$, but can simply regard it as a fixed set of sets (social spheres) $R$, defined on the domain of persons, $U$.
6. Q: What constraints on the system of spheres are justified given our goal of representing social rank?

($U$ for *universe*)

Each sphere is, as it were, an exclusive social club, and if your friend is a member but you are not, then your friend ranks more highly than you do.

Call $R$ a *set of social spheres* if meets the following three conditions:

1) Nested: for $T, S \in R$, either $S \subseteq T$ or $T \subseteq S$
2) Unions: if $\bigcup S$ is the union of a set of spheres in the system $R$, then $\bigcup S \in R$.
3) Intersections: if $\bigcap S$ is a non-empty intersection of a set of spheres in the system $R$, then $\bigcap S \in R$.

These are three of the four constraints Lewis used—we've dropped Centering.

Call $M$ a *social ranking model* if it is a triple $(U, R, I)$ in which $U$ is a set of individuals, $R$ a set of social spheres, and $I$ an interpretation function.

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## *Applications*

Why would we want to go and do something like that?
Lewis' systems of spheres can be used to:

1) give truth-conditions for modal claims like $\Box p$ and $\Diamond p$
2) (as we've already seen) give truth-conditions for counterfactual claims like $A \mathrel{\Box\!\!\rightarrow} B$
3) identify counterfactual fallacies, e.g. strengthening the antecedent, transitivity, and contraposition.
4) explain the pragmatics of counterfactual utterances

My plan for the rest of the talk: look at each of these four applications see whether we can do something similar with social spheres.

## 1. *Modal Operators and Hierarchical Quantifiers*

> "Many men of course became extremely rich, but this was perfectly natural and nothing to be ashamed of because no one was really poor—at least no one worth speaking of." (Adams' *Hitchhikers*)

— Let's start simple. Lewis' systems of spheres continue to allow us to interpret the unary modal operators □ and ◇.

— □$A$ is true at a world $w$ just in case $A$ is true throughout $w$'s sphere of accessibility.

— But *which* sphere of accessibility? $w$ is at the center of a nested *set* of spheres.

— 2 options:

1. a *set* of subscripted operators,
2. a single operator disambiguated by conversational context.

Let's go with 2:

| |
|---|
| □$A$ is true at a world $w$ just in case $A$ is true throughout the sphere of accessibility determined by $\$_w$ and the conversational context $c$.<br>◇$A$ is true at a world $w$ just in case $A$ is true at at least one world in the sphere of accessibility determined by $\$_w$ and the conversational context $c$. |

— Accommodation.

— Problem: Worlds are things relative to which a sentence gets a truth value, but persons are not.

— Solution: The model-theoretic analogue for the truth of a sentence when the object is an individual rather than a world is *satisfaction* of a 1-place predicate. And the syntactically appropriate expression for quantifying over individuals isn't a sentential operator, but a unary first-order *quantifier:* a device that, syntactically, attaches to predicates to form sentences, and—model theoretically—to sets to give truth-values.

— Some truth-clauses for monadic social quantifiers:

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|---|
| $\forall\alpha A$ is true just in case $A$ is satisfied by every member of the social sphere $S$ determined by $R$ and the conversational context $c$.<br>$\exists\alpha A$ is true just in case $A$ is satisfied by at least one member of the social sphere $S$ determined by $R$ and the conversational context $c$.<br>$\mathtt{Most}\alpha A$ is true just in case $card(\|A\|_S) > card(\|U - A\|_S$ (where $S$ is determined by $R$ and the context, $c$.) |

It's impossible for you to get to New York by 5pm; you've got that dinner in Carrboro tonight

It's impossible for you to get to New York by 5pm; they just closed the airport.

It's impossible for you to get to New York by 5pm; the fastest any extant vehicle can go is 2200 mph.

It's impossible for you to get to New York by 5pm; it's 5.01pm now.

**Most** people have seen *Hamilton*.

I'm not going to get stuck in this job; I'm going to be **someone**.

You really have to be **someone** to be asked to Buckingham Palace.

**Nobody** needs to see another *Nutcracker/Carmen/Spiderman.*

**Everyone** from the Manor goes to London for the season.

**Nobody** in this city can manage without a cleaner/personal trainer/nanny/driver/life-coach.

**Everyone** summered in the Hamptons.

**Everyone** should own a gun.

**All** men are created equal.

↖ Truth conditions for *unary* social quantifiers. Quantifier analogues of contextually determined □ and ◇)

Here 'is true' is short for 'is true in the model $M$'

## 2. *Counterfactuals and Variable Binary Social Quantifiers*

**Syntax**: If $A$ and $B$ are formulas, and $\alpha$ is a variable over individuals, then $\forall^2\alpha(A,B)$ is a formula.[1]
**Interpretation**: $\forall^2\alpha(A,B)$ is true iff $|A| \subseteq |B|$.

← Truth conditions for *strict* binary universal quantifier $\forall^2$ (Quantifier analogue of strict conditional: $A \prec\!\!\!\!\!\!-\!\!\!3\ B$ or $\Box(A \rightarrow B)$.)

[1] Some terminology: We will call the formula in the $A$-position the *restrictor*, and the formula in the $B$-position the *scope.* The set of individuals which satisfy the restrictor formula, $|A|$ (when assigned to the variable $\alpha$) is then the *restrictor set* and the set of individuals which satisfy the scope formula, $|B|$, is the *scope set*. $X_s$ means the intersection of a set $X$ with the sphere $S$, (X restricted to S) and so e.g. $|A|_S$ is intersection of the restrictor set with the sphere S.

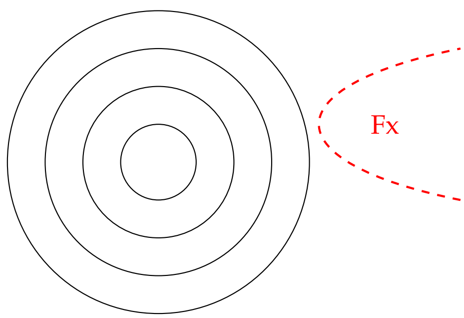
### *Variable Binary Social Quantifiers*

$\forall^{2v}\alpha(A,B)$ true if and only if either

1. no individual in any sphere in $R$ satisfies $A$, or
2. some individual in some sphere $S \in R$ does satisfy $A$, and $|A|_S \subseteq |B|_S$.

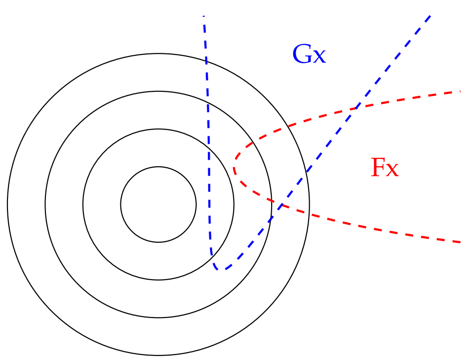
← truth-conditions for *variable* universal binary quantifier, $\forall^{2v}$. Analogue of $\Box\!\!\rightarrow$

### *Vacuous Truth*




$\forall^{2v}x(Fx, Gx)$—true
$\forall^{2v}x(Fx, \neg Gx)$—also true
e.g. "All werewolves are hungry."

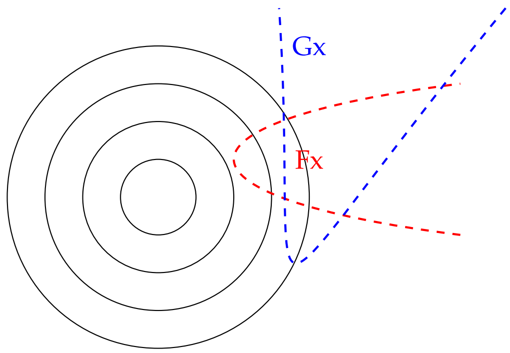
### *Non-vacuous Truth*




$\forall^{2v}x(Fx, Gx)$—true
$\forall^{2v}x(Fx, \neg Gx)$—false
e.g. "All women are permitted to vote."

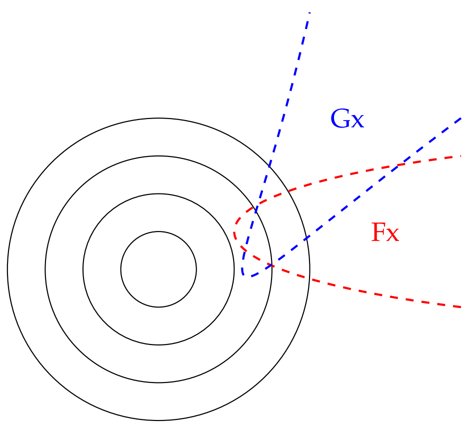
### *Falsity; Opposite True*

$\forall^{2v}x(Fx, Gx)$—false
$\forall^{2v}x(Fx, \neg Gx)$—true
e.g. "All women are safe from harassment."




### *Falsity; Opposite False*

$\forall^{2v}x(Fx, Gx)$—false
$\forall^{2v}x(Fx, \neg Gx)$—false
e.g. "All women like dogs."




## 3. *Fallacies*

### *Strengthening the Antecedent (of a counterfactual)*

$$\frac{A \mathrel{\Box\!\!\rightarrow} B}{(A \wedge C) \mathrel{\Box\!\!\rightarrow} B}$$

$$\frac{\text{Kangaroos don't have tails} \mathrel{\Box\!\!\rightarrow} \text{they fall over.}}{(\text{ Kangaroos don't have tails} \wedge \text{they use crutches }) \mathrel{\Box\!\!\rightarrow} \text{they fall over.}}$$

### *Strengthening the Restrictor (of a quantifier)*

$$\frac{\forall^{2v}\alpha(A, B)}{\forall^{2v}\alpha(A \wedge C, B)}$$

$$\frac{\forall^{2v}x((\text{Australian}(x) \wedge \text{ Woman(x))(AbleToVote(x))}}{\forall^{2v}x((\text{Australian}(x) \wedge \text{ Woman(x)} \wedge \text{Indigenous(x)) (AbleToVote(x))}}$$

### *Counterfactual Transitivity*

$$\frac{A \mathrel{\Box\!\!\rightarrow} B \quad B \mathrel{\Box\!\!\rightarrow} C}{A \mathrel{\Box\!\!\rightarrow} C}$$

If J.Edgar Hoover had been born a Russian, he would have been a communist.
If J. Edgar Hoover had been a communist, he would have been a traitor.
___
If J. Edgar Hoover had been a Russian, he would have been a traitor.

### *Quantifier Transitivity*

$$\frac{\forall^{2v}\alpha(A,B) \quad \forall^{2v}\alpha(B,C)}{\forall^{2v}\alpha(A,C)}$$

All ASL-speakers gesticulate while speaking.
Everyone who gesticulates while speaking should take a course on body language.
___
All ASL-speakers should take a course on body language.

### *Contraposition (counterfactuals)*

$$\frac{A \mathrel{\Box\!\!\rightarrow} B}{\neg B \mathrel{\Box\!\!\rightarrow} \neg A} \qquad \frac{\neg B \mathrel{\Box\!\!\rightarrow} \neg A}{A \mathrel{\Box\!\!\rightarrow} B}$$

If Boris had gone to the party, Olga would still have gone.
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If Olga hadn't gone to the party, Boris would still not have gone.

Boris is avoiding the party to avoid Olga.

### *Quantifier contraposition*

$$\frac{\forall^{2v}\alpha(A,B)}{\forall^{2v}\alpha(\neg B,\neg A)}$$

$$\frac{\forall^{2v}\alpha(\neg B,\neg A)}{\forall^{2v}\alpha(A,B)}$$

All black people are permitted to vote.
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Anyone not permitted to vote is not black.

$$\frac{\forall^{2v}x(Bx, Vx)}{\forall^{2v}x(\neg Vx, \neg Bx)}$$

## 4. *Pragmatic Phenomena*

### *Accommodation and Similarity between Worlds*

(1) a) If Caesar had been in command, he would have used the atomic bomb.

b) If Caesar had been in command, he would have used catapults.

"In dealing with Quine's opposed counterfactuals about Caesar, context must of course be consulted somehow. [...] I could [...] call on context [...] to resolve part of the relation of comparative similarity in a way favorable to the truth of one counterfactual or the other. In one context, we may attach great importance to the similarities and differences in respect of Caesar's character and in respect of regularities concerning the knowledge of weapons common to commanders in Korea. In another context we may attach less importance to these similarities and differences, and more importance to similarities and differences in respect of Caesar's own knowledge of weapons. The first context

> resolves the vagueness of comparative similarity in such a way that some worlds with a modernized Caesar in command come out closer to our world than any with an unmodernised Caesar. It thereby makes the first counterfactual true. The second context resolves the vagueness in the opposite direction, making the second counterfactual true. Other contexts might resolve the vagueness in other ways." (67) [2]

[2] Lewis, D. (1973). *Counterfactuals*. Blackwell, Oxford

> "If at time *t* something is said that requires component $s_n$ of conversational score to have a value in the range *r* if what is said is to be true, or otherwise acceptable; and if $s_n$ does not have a range in the value *r* just before *t*; and if such and such further conditions hold; then at *t* the score component $s_n$ takes some value in the range *r*." (347) [3]

[3] Lewis, D. (1979). Scorekeeping in a language game. *Journal of Philosophical Logic*, 8:339–359

### *Accommodation and Social Ranking*

(2) Rufus: "You ought to call me 'master'—after all, you want me to call you 'black'. "

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