Minutes of the discussion of the UConn Logic Group meeting
September 19, 2008

provided by Marcus Rossberg <marcus.rossberg@uconn.edu>

The session contained two presentations:

1. Colin on the first-order version of $Ł_{\aleph}$
2. Reed on the Effective Completeness Theorem for classical first-order logic

The following points were raised in the discussion:

1. On Colin's presentation

   a. Colin mentioned in his presentation that there was no axiomatisation the first-order version of $Ł_{\aleph}$. Reed asked there was merely none *yet*, or whether $Ł_{\aleph}$ is in fact non-axiomatisable.

   No one knew the answer to this during the discussion, but Marcus later reported that $Ł_{\aleph}$ is indeed non-axiomatisable, if the valuations are defined on $[0, 1]$, as proved by Scarpellini [2]. (As Colin pointed out last week, logical truth of the propositional fragment is axiomatisable; logical consequence however is not.)

   If the logic is defined on multi-valued algebras instead, however, the resulting consequence relation is axiomatisable; see Hájek [1], §3, for details.

   b. Jc and Reed remarked that Colin's proof of the third fact on page 4 of his handout established something a lot stronger than the fact stated (the fact is entailed, of course).

   c. Reed wondered whether $Ł_{\aleph}$ was compact.

   d. Jc remarked that in $Ł_{\aleph}$ you can have non-trivial naïve set theory, where:

   - *naïve set theory* is the (classically inconsistent) set theory that contains an unrestricted set comprehension principle: $\exists\alpha\forall x(x \in \alpha \equiv \varphi(x))$;
   - *triviality* is the paraconsistenists' analogue of inconsistency, as it were: a theory is trivial iff for every sentence φ (of the relevant language), both φ and $\neg\varphi$ are in the theory. Triviality is classically entailed by inconsistency, owing to the principle *ex falso quodlibet*, that a contradiction entails everything (a.k.a. "explosion", in some circles...). Where *ex falso quodlibet* is given up (in relevant and paraconsistent logics, for instance) a theory can be inconsistent, i.e. contain both φ and $\neg\varphi$ for *some* sentence φ, without being trivial.

   Jc also mentioned a proof by White that this system is not only non-trivial, but also consistent. The proof can be found in [3].

2. On Reed's presentation

   a. Lionel enquire whether the "in addition" was needed in the definition of the decidability of an $\mathcal{L}$-structure on Reed's handout, page 2, 10th line from the bottom. It seems that the existence of an algorithm suffices for computability and decidability; or, in different words, the latter entails the former.

      Reed confirmed Lionel's suspicion.

References

[1] Petr Hájek. *Metamathematics of Fuzzy Logic*. Kluwer, Dordrecht, 1998.

[2] Bruno Scarpellini. Die Nichtaxiomatisierbarkeit des unendlichwertigen Prädikatenkalküls von Łukasiewicz. *Journal of Symbolic Logic*, 27:159–170, 1962.

[3] Richard B. White. The consistency of the axiom of comprehension in the infinite-valued predicate logic of Łukasiewicz. *Journal of Philosophical Logic*, 8:509–534, 1978.