Minutes of the discussion of the UConn Logic Group meeting
September 19, 2008

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The session contained two presentations:

1. Colin on the first-order version of $L_{\mathbb{N}}$
2. Reed on the Effective Completeness Theorem for classical first-order logic

The following points were raised in the discussion:

1. On Colin’s presentation
   
   a. Colin mentioned in his presentation that there was no axiomatisation the first-order version of $L_{\mathbb{N}}$. Reed asked there was merely none yet, or whether $L_{\mathbb{N}}$ is in fact non-axiomatisable.

   No one knew the answer to this during the discussion, but Marcus later reported that $L_{\mathbb{N}}$ is indeed non-axiomatisable, if the valuations are defined on $[0,1]$, as proved by Scarpellini [2]. (As Colin pointed out last week, logical truth of the propositional fragment is axiomatisable; logical consequence however is not.)

   If the logic is defined on multi-valued algebras instead, however, the resulting consequence relation is axiomatisable; see Hájek [1], §3, for details.

   b. Jc and Reed remarked that Colin’s proof of the third fact on page 4 of his handout established something a lot stronger than the fact stated (the fact is entailed, of course).

   c. Reed wondered whether $L_{\mathbb{N}}$ was compact.

   d. Jc remarked that in $L_{\mathbb{N}}$ you can have non-trivial naïve set theory, where:

   • **naïve set theory** is the (classically inconsistent) set theory that contains an unrestricted set comprehension principle: $\exists \alpha \forall x (x \in \alpha \equiv \varphi(x))$;

   • **triviality** is the paraconsistentists’ analogue of inconsistency, as it were: a theory is trivial iff for every sentence $\varphi$ (of the relevant language), both $\varphi$ and $\neg \varphi$ are in the theory. Triviality is classically entailed by inconsistency, owing to the principle *ex falso quodlibet*, that a contradiction entails everything (a.k.a. “explosion”, in some circles…). Where *ex falso quodlibet* is given up (in relevant and paraconsistent logics, for instance) a theory can be inconsistent, i.e. contain both $\varphi$ and $\neg \varphi$ for some sentence $\varphi$, without being trivial.

   Jc also mentioned a proof by White that this system is not only non-trivial, but also consistent. The proof can be found in [3].
2. On Reed’s presentation

   a. Lionel enquire whether the “in addition” was needed in the definition of the decidability of an $\mathcal{L}$-structure on Reed’s handout, page 2, 10th line from the bottom. It seems that the existence of an algorithm suffices for computability and decidability; or, in different words, the latter entails the former.
   
   Reed confirmed Lionel’s suspicion.

References

