Negation in Natural Language

(1) \[\llbracket \text{not}_t \rrbracket = \begin{bmatrix} 1 & \rightarrow & 0 \\ 0 & \rightarrow & 1 \end{bmatrix}\]

(2) **Recursive definition of not**
If not is defined for type \(\beta\), then
\[\llbracket \text{not}_{<\alpha,j>} \rrbracket(f) = \lambda x_e.\llbracket \text{not}_t \rrbracket(f(x))\]

(3) For example, predicate negation:
   a. \[\llbracket \text{not}_{<e,t>} \rrbracket = \lambda f_{<e,t>}. \lambda x_e.\llbracket \text{not}_t \rrbracket(f(x))\]
   b. \([S \text{ Bill did } [VP \text{ not run }]]\]

(4) \[\llbracket \text{NP not}_{<e,t,s>} \rrbracket \quad \llbracket \text{NP every student } \rrbracket \text{ smokes}\]

(5) Given (2), not is defined for all 'conjoinable' types.
   Conjoinable types are types that 'end in t.'

Grammatical correlates of negation: Klima 1964

(6) a. Publishers reject suggestions, writers will *(not) accept them, either.
   b. Publishers always reject suggestions,
      writers will never/*sometimes accept them, either.
   c. All publishers reject suggestions, no/*some writers accept them, either.

(7) Claim: \(\text{no} = \text{not} + \text{some}\)
    \(\text{never} = \text{not} + \text{sometimes}\)

(8) **Either** is acceptable in a sentence S only if S contains an instance of not.

(9) a. Bill does not think he will ever go to France.
   b. Sue never thinks she will ever go to France.
   c. No/*some student thinks he will ever go to France.
   d. Only Bill thinks he will ever go to France.

(10) Only writers reject suggestions. *Only writers reject compliments, either.

(11) **Ever** is acceptable in sentences that do not contain an instance of not.
   Klima’s solution: **Ever** is acceptable in S only if
   S contain a [+Affective] expression.

(12) Which expressions are [+Affective]?
1. A semantic generalization of the notion of negativity/affectivity: Ladusaw 1979

(13) A function $f$ is **downward monotone** (DM) iff for any $x, y$ in $f$'s domain such that $x \leq y$, $f(y) \leq f(x)$

(14) $\lnot$ is downward monotone.

(15) Downward monotonicity is a plausible analysis of Klima's Affectivity [See Yael's portion for an application to *only*]

(16) a. No student has ever run.
    b. *Every student has ever run.*

(17) a. $\lceil\text{no student}\rceil = \lambda f_{<e>}. \text{there is no } x \text{ s.t. } x \text { is a student and } f(x)=1$
    b. $\lceil\text{every student}\rceil = \lambda f_{<e>}. \text{for all } x \text{ s.t. } x \text { is a student, } f(x)=1$

(18) (17)a is DM, (17)b is not DM.
    a. $\lceil\text{run quickly}\rceil \leq \lceil\text{run}\rceil$ ($\lceil\text{run}\rceil = \lambda f_{e}. x \text{ runs}$)
    b. $\lceil\text{no student runs}\rceil \leq \lceil\text{no student runs quickly}\rceil$
    c. $\lceil\text{every student runs}\rceil \geq \lceil\text{every student runs quickly}\rceil$

(19) Some more examples:
    a. Few students have ever passed.
    b. *Some students have ever passed.*

If affectivity can be eliminated in favor of DM, how do we account for distinction between *ever* and *either*?


(20) Grade negative function in terms of strength by amount of deMorgan’s Laws they validate.

(21) a. Law 1: $f(a) \lor f(b) \Rightarrow f(a \land b)$
    b. Law 2: $f(a \lor b) \Rightarrow f(a) \land f(b)$
    c. Law 3: $f(a) \land f(b) \Rightarrow f(a \lor b)$
    d. Law 4: $f(a \land b) \Rightarrow f(a) \lor f(b)$

(22) $\lnot$ satisfies all four laws; so, it is full strength negation. Zwarts calls all functions that satisfy all four laws **antimorphic**.

(23) A function that satisfies (at least) Laws 1-3 is **anti-additive** (AA). $\lceil\text{no student}\rceil$ is not antimorphic, but is anti-additive. Same for *never.*
(24) Satisfying Laws 1 and 2 is equivalent to being downward monotone. \([\text{fewer than 4 students}]\) is downward monotone, but not anti-additive.

(25) A function that satisfies (at least) Laws 1, 2 and 4 is \textit{anti-multiplicative}. \([\text{not every student}]\) is anti-multiplicative. No grammatical phenomenon correlates with anti-multiplicativity.

(26)

\begin{align*}
\text{not} & \quad \text{antimorphic} \\
\downarrow & \\
\text{never, no students} & \quad \text{anti-additive} \\
\downarrow & \\
\text{rarely, fewer than 4 students} & \quad \text{downward monotone}
\end{align*}

(27) \textbf{Either} and \textbf{ever} are \textit{Negative Polarity Items} that require licensors of different strengths. 
  a. \textbf{Ever} must bear a special structural relation with a DM function.
  b. \textbf{Either} must bear a special structural relation with a AA function.

3. Klima again

(28) This is a semantic reconstruction of Klima’s distinction, but is there some independent evidence that Klima was right that, e.g., \textit{no} = \textit{not}+\textit{some}.

(29) \textbf{Scope splitting} \\
The department need fire no professors.

(30) Reading 1: No professors x are s.t. the department needs to fire x.

(31) Reading 2: The department does not need to fire any professors.

(32) Logical Form of (29) on Reading 2:

\[
[ \text{not} [ \text{need} [ \text{the department fires some professors} ] ] ]
\]

[Last time we saw that NPIs are licensed by Downward Entailing functions (*\textit{John has ever been to Paris} vs. It isn’t true that \textit{John has ever been to Paris}).]

\textbf{The Conditionals Puzzle}

Two seemingly conflicting properties of conditionals:

NPIs are licensed in the antecedent.

(1) a. If I see anyone on campus today, I’ll know it’s not Sunday.
   b. If John has \textit{ever} been to Paris, he is no stranger to fine cuisine.
No strengthening of the antecedent
(2) a. If I strike a match, there will be fire. =/=>
b. If I strike a wet match, there will be fire.

I. The strict conditional analysis
(3) $[[\text{if-then}]]^f_{R}(p)(q) = \text{True}$ iff $\forall w' \in f(w): p(w') = \text{True} \rightarrow q(w') = \text{True}.$
   (p and q are propositions, and f an accessibility function)

Predicts NPI-licensing in the antecedent clause (cf. Every student who has ever been to Paris is no stranger to fine cuisine), but doesn’t explain why there is no strengthening of the antecedent (cf. Every student left => Every French student left).

II. The non-monotonic analysis (Stalnaker, Lewis, Kratzer, von Fintel)
(4) a. $w <_S w'$ iff $\{p \in S: p(w) = \text{True}\} \supset \{p \in S: p(w') = \text{True}\}$
   b. Max$(X, S) := \{w \in X: \neg \exists w' \in X: w' <_S w\}$
   c. $[[\text{if-then}]]^{f,g}_{R}(p)(q) = \text{True}$ iff $\text{Max}(\{w': p(w') = \text{True}\} \cap f(w), R(w)) \subseteq \{w': q(w') = \text{True}\}.$
      (R is an ordering source)

Predicts no strengthening of the antecedent, but not licensing of NPIs.

Resolving the conflict: The Heim/von Fintel Analysis
1st Ingredient: Strawson DE
   c. Only John ate vegetables. =/=> d. Only John ate kale.

BUT:
(6) a. John didn’t eat any vegetables.
   b. Only John ate any vegetables.

(7) Classical entailment:
   (i) If A and B are of type t, A entails B iff A is False or B is True.
   (ii) If A and B are of type $<\sigma, t>$, A entails B iff for all x of type $\sigma$, $A(x)$ entails $B(x)$.

(8) Strawson downward entailment: A function f is SDE iff for all A and B such that A entails B and f(A) is defined: f(B) entails f(A).

(9) $[[\text{only}]](x)(P)$ is defined only if $P(x) = \text{True}$.
   Whenever defined, $[[\text{only}]](x)(P) = \text{True}$ iff For all $y \neq x$, $P(y) = \text{False}$.

So $[[\text{only}]]$ is not DE on P in the classical sense, but it is SDE on P.

(10) New Condition on NPI-licensing: NPIs are licensed by SDE functions.

2nd ingredient: a revised semantics for if-then
(11) $[[\text{if-then}]]^{f,g}_{R}(p)(q)$ is defined only if (i) and (ii) hold:
   (i) f is admissible relative to R and g;  (Admissibility presupposition)
(ii) \( \exists w' \in f(w) : p(w') = True \) (Compatibility presupposition)

Whenever defined, \([if-then]_{fg,R}^w(p)(q) = True \) iff
\[ \forall w' \in f(w) : p(w') = True \rightarrow q(w') = True. \]

(12) \( f \) is admissible relative to \( R \) and \( g \) iff for any world \( w \),
(i) \( \forall n > 0 : \ (\exists w' \in Max_n(g(w),R(w)) : w' \in f(w)) \rightarrow (\forall w' \in Max_n(g(w),R(w)) : w' \in f(w)); \) and
(ii) \( \forall n > 1 : \ Max_n(g(w),R(w)) \subseteq f(w) \rightarrow Max_{n-1}(g(w),R(w)) \subseteq f(w). \)

(a) \( Max_1(X,S) := Max(X,S); \)
(b) \( Max_m(X,S) := Max((X - \{Max_1(X,S), ..., Max_{m-1}(X,S)\}), S), \) for any \( m > 1. \)

Consequences: The Compatibility presupposition guarantees no strengthening of the antecedent. It also makes \([if-then] \) SDE on \( p \).

Contexts are updated to satisfy the Compatibility presupposition.
(13) a. If the USA threw its weapons into the sea tomorrow, there would be war;
but if all the nuclear powers threw their weapons into the sea tomorrow, there would be peace.

b. ??If all the nuclear powers threw their weapons into the sea tomorrow, there would be peace; but if the USA threw its weapons into the sea tomorrow, there would be war.

Rather than show that the context doesn’t change (which was the point Lewis wanted to make based on (13a)), the contrast between (13a) and (13b) (attributed by von Fintel to Irene Heim) only shows that changing the context cannot involve shrinking the "horizon".

Examples of remaining problems
(14) a. If we are on Route 195, then we might be in Storrs.
   b. If we ever get to NY, we might try to get tickets for the opera.
(15) a. John wonders which students have ever taken Semantics I.
   b. (##)John knows which students have ever taken Semantics I.
   c. #It surprised John which students had ever taken Semantics I.