Lionel Shapiro (Sept. 17, 2010)

Deflating Logical Consequence

1. Big picture

What is the nature of truth?

Rival approaches: correspondence to reality, justifiability/assertability.

What is the nature of logical consequence?

Rival approaches: preservation of truth under all interpretations, derivability according to some system of deduction rules. (Stewart 'The Guru' Shapiro: "In broad terms, there are two different approaches to consequence: deductive and semantic.")

But there's a major disanalogy between the debates:

- Since Quine (1970), there has been a prominent third option in the truth debate: "deflationists" dismiss the question the traditional answers seek to answer.
- My question: is there room for a parallel position regarding logical consequence?

2. One kind of deflationism about truth

The essential role of the predicate 'is true' is to allow us to simulate quantification *into sentence position* by generalizing instead *over objects*:

Every sentence of form '$p \vee \sim p$' is true.
Some of what he said is not true.

How does the truth predicate manage to serve this function?

(T-Int) $\dfrac{p}{\text{`}p\text{' is true}}$ (T-Elim) $\dfrac{\text{`}p\text{' is true}}{p}$

<u>A deflationary argument</u> (implicit in Quine):

T1. If it weren't for the need to express a certain kind of generality, we would have no need for the predicate 'is true'. Instead of predicating 'is true' of a sentence, we could employ the sentence itself.

T2. To explain how 'is true' allows us to express the kind of generality in question, we need *only* make use of to the predicate's logical features, namely the T-rules.

T3. There is no reason to think that the T-rules require 'is true' to express a property whose nature is amenable to substantive characterization.

T4. From T1, T2, and T3, it follows that there is no reason to think that in order to understand how 'is true' serves the function that is its *raison d'être*, we must take this predicate to express a property whose nature is amenable to substantive characterization.

T5. Hence we have no reason to hold that 'is true' expresses a property whose nature is amenable to substantive characterization.

3. Why not say the same about logical consequence?

The essential role of the predicate 'S_1 has S_2 as a logical consequence' lies in enabling generalizations:

Every sentence of form '$p \wedge q$' has the respective 'p' as a consequence.
He denied a consequence of something she said.

How does the consequence predicate manage to serve this function?

(C-Intro) $\dfrac{\text{That } p \text{ entails that } q}{\text{`}p\text{' has `}q\text{' as a consequence}}$ (C-Elim) $\dfrac{\text{`}p\text{' has `}q\text{' as a consequence}}{\text{That } p \text{ entails that } q}$

At least for the present, we can handle multiple-premise consequence with conjunction.

A deflationary argument:

C1. If it weren't for the need to express a certain kind of generality, we would have no need for the predicate 'is a consequence of'. In place of 'S_1 is a consequence of S_2', we could employ sentences S_1 and S_2 themselves, joined by a suitable sentential connective.

C2. To explain how 'is a consequence of' allows us to express the kind of generality in question, we need *only* make use of the predicate's logical features, namely the C-rules.

C3. There is no reason to think that the C-rules require 'is a consequence of' to express a relation whose nature is amenable to substantive characterization.

C4. From C1, C2, and C3, it follows that there is no reason to think that in order to understand how 'is a consequence of' serves the function that is its *raison d'être*, we must take this predicate to express a relation whose nature is amenable to substantive characterization.

C5. Hence we have no reason to hold that 'is a consequence of' expresses a relation whose nature is amenable to substantive characterization.

4. Philosophical objections

The usual objection:

Doesn't this "deflationism" just transfer the substantive issues from *consequence* to *entailment*?

My short answer:

If this objection were good, it would work just as well against deflationism about falsity. According to the objector, deflationism about falsity would merely transfer substantive issues from *falsity* to *negation*. (Yet I don't know of any deflationist about truth who rejects deflationism about falsity.)

More specific objections:

(a) Won't "deflating consequence" leave a question about the nature of entailment facts?
Question: Does deflating falsity leave a question about the nature of negative facts?

(b) Won't the debate about "logical pluralism" persist?
Answer: Sure, just like debates about whether there are multiple negation connectives.

(c) Won't there be substantive questions about the connection between entailment and inference?
Answer: Sure, just like questions about the connection between negation and rejection or denial.

5. Consequences for logic

Whether or not we draw *deflationary morals*, it's plausible that the consequence relation has the expressive role I have described. But two deflationists (H. Field and J. Beall) argue that no relation can have this role. Are they right?

Curry's paradox

Let $Cons(x, y)$ abbreviate our consequence predicate and $\rightarrow$ our entailment connective.
Let P be an arbitrary sentence, and suppose we have a sentence K equivalent to $Cons(\langle K\rangle, \langle P\rangle)$.
Then we have the following apparent natural-deduction proof of P.

Assumptions		
1	(1) K	Ass
1	(2) $Cons(\langle K\rangle, \langle P\rangle)$	1, Equivalence
1	(3) $K \rightarrow P$	*2,* C-Elim
1	(4) P	1, 3 $\rightarrow$Elim (modus ponens)
	(5) $K \rightarrow P$	1, 4 $\rightarrow$Intro (conditional proof)
	(6) $Cons(\langle K\rangle, \langle P\rangle)$	5, C-Intro
	(7) K	6, Equivalence
	(8) P	5, 7 $\rightarrow$Elim (modus ponens)

How to block this derivation?

- For their own proposed conditionals, Field and Beall reject $\rightarrow$Intro.
- But if the arrow is an entailment connective, this looks unacceptable: "We clearly want to be able to assert $A \rightarrow B$ whenever there exists a deduction of B from A" (Anderson and Belnap).
- So must we reject $\rightarrow$Elim instead? Isn't this "analytically part of what [entailment] is" (Priest)? Well, it depends what counts as preserving $\rightarrow$Elim.

<u>Substructural natural deduction</u> (S.L. Read, based on G. Mints and J.M. Dunn in 1970s)
Here X and Y are "structured collections" of indices, structured using ',' and ';'.

Assumptions			*Assumptions*			*Assumptions*		
X	$\boldsymbol{A}$		X	$\boldsymbol{A \rightarrow B}$		Y	$\boldsymbol{A}$	
Y	$\boldsymbol{B}$		Y	$\boldsymbol{A}$		X; Y	$\boldsymbol{B}$	
X, Y	$\boldsymbol{A \wedge B}$	$\wedge$Intro	X; Y	$\boldsymbol{B}$	$\rightarrow$Elim	X	$\boldsymbol{A \rightarrow B}$	$\rightarrow$Intro

In place of line (4) above, we can only write

1; 1 (4*) P 1, 3 $\rightarrow$Elim

We would need a an additional "structural" rule to get from this to

1 (4) P 4 Weak Contraction

Without this rule, (4*) only gets us

1 (5*) $K \rightarrow P$ 1, 4* $\rightarrow$Intro

<u>Field's complaint</u>
Substructural deduction systems without contraction are "hard to get my head around."
That's because he interprets the step to (4*) in terms of *conditional assertability*:

> Given that we may assert $K \rightarrow P$ on the assumption of K, and that we may assert K on the assumption of K, we may assert P on the <u>double</u> assumption of K. [???]

But we can instead interpret the step as registering an *entailment*

> That $K \rightarrow P$ is a consequence of K, and that K is a consequence of K, together entail that it's a consequence of K that P is a consequence of K.

And it seems perfectly sensible to regard this way of using 'consequence'-talk as reflecting the following theorem of contraction-free logics.

$$(K \rightarrow (K \rightarrow P)) \rightarrow ((K \rightarrow K) \rightarrow (K \rightarrow (K \rightarrow P))).$$

<u>Consequence and truth-preservation</u>
Say we endorse both these principles:

(i) Consequence preserves truth, where 'preservation' is spelled out using $\rightarrow$.

(ii) B is a consequence of $(A \rightarrow B) \wedge A$.

As Field and Beall point out, this commits us (given natural assumptions about truth) to the following recipe for Curry's paradox:

(iii) $((A \rightarrow B) \wedge A) \rightarrow B$

They reject (i), but that's not an option if we want C-Elim. In that case, we must reject (ii).

My challenge for Field and Beall:

- They are willing to revise classical logic, by rejecting the Law of Excluded Middle (Field) or Explosion (Beall), in order to preserve the expressive utility of a predicate obeying the T-rules.
- Why are they unwilling to admit violations of (ii) in order to preserve the expressive utility of a predicate obeying the C-rules?